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Joint work with the COSMOGRAV GROUP at SSM

Exploring metric departures from Schwarzschild black hole geometry

Motivations of my talk

- ❖ We consider a general static and spherically symmetric metric in $f(R)=R^k$

$$ds^2 = -e^{\nu(r)} dt^2 + e^{\lambda(r)} dr^2 + r^2(d\theta^2 + \sin^2 \theta d\varphi^2)$$

- ❖ General solution

$$e^{\nu} = r^{\frac{2\epsilon(1+2\epsilon)}{1-\epsilon}} + \frac{C_1}{r^{\frac{1-4\epsilon}{1-\epsilon}}},$$

$$C_1 \in \mathbb{R}$$

$$|\epsilon| \ll 1 \quad e^{\lambda} = \left\{ \left[\frac{(1-\epsilon)^2}{(1-2\epsilon+4\epsilon^2)[1-2\epsilon(1+\epsilon)]} \right] \left(1 + \frac{C_1}{r^{\frac{1-2\epsilon+4\epsilon^2}{1-\epsilon}}} \right) \right\}^{-1}.$$

- ❖ Particular solution

$$\left. \begin{array}{l} \epsilon = -0.01 \\ C_1 = -r_0^{1.01} \\ r_h \equiv r_0 = 2.01M \end{array} \right\} (M=1) \quad \left. \begin{array}{l} e^{\nu(r)} = \frac{1}{r^{0.02}} - \frac{2.02}{r^{1.03}}, \\ e^{\lambda(r)} = \frac{1.02}{1 - \frac{2.02}{r^{1.01}}}. \end{array} \right\} r_{\text{obs}} = 10^{10}M \quad \Rightarrow \quad \left. \begin{array}{l} \lim_{r \rightarrow r_{\text{obs}}} e^{\nu(r)} = 0.64, \\ \lim_{r \rightarrow r_{\text{obs}}} e^{\lambda(r)} = 1.02. \end{array} \right\}$$

What are the astrophysical strategies we can employ in order to distinguish between Schwarzschild geometry and our metric?

Proposed approach

Employing the combination of different astrophysical strategies depending on the context under study

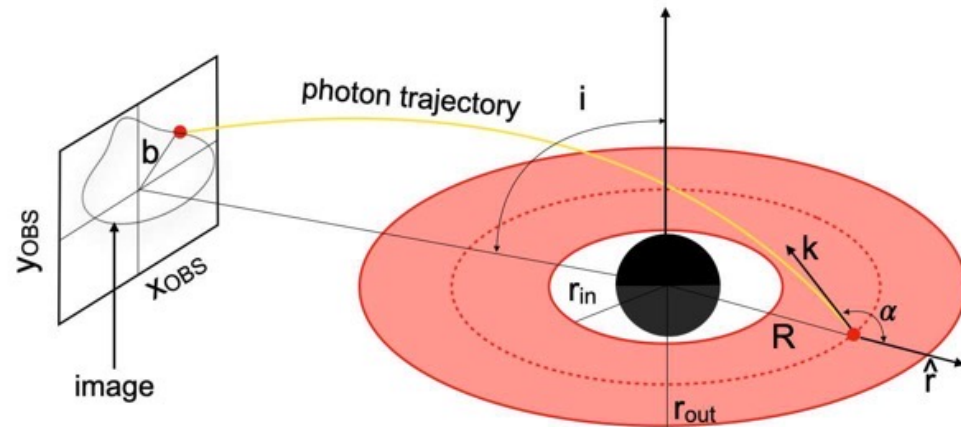
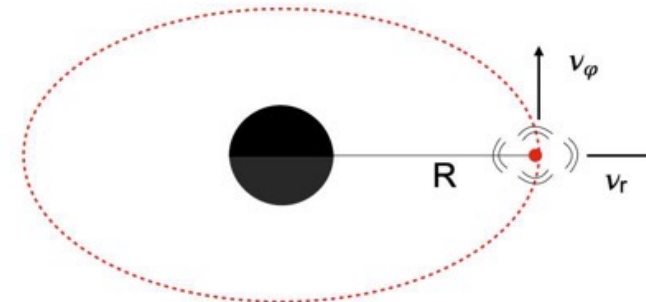
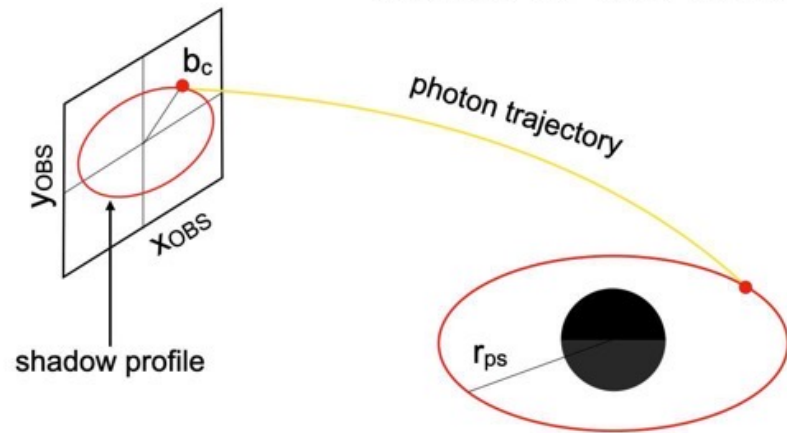


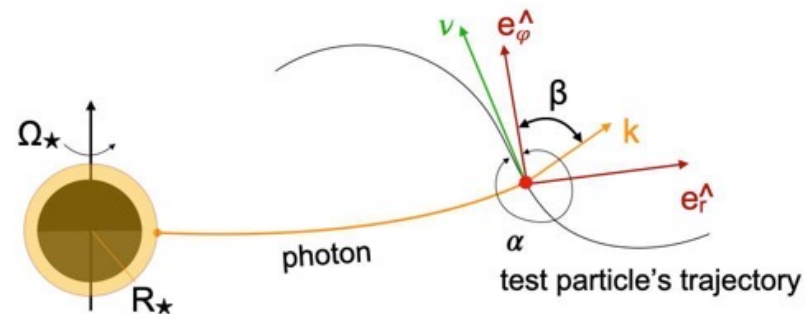
IMAGE OF A BLACK HOLE



EPICYCLIC FREQUENCIES



SHADOW OF A BLACK HOLE



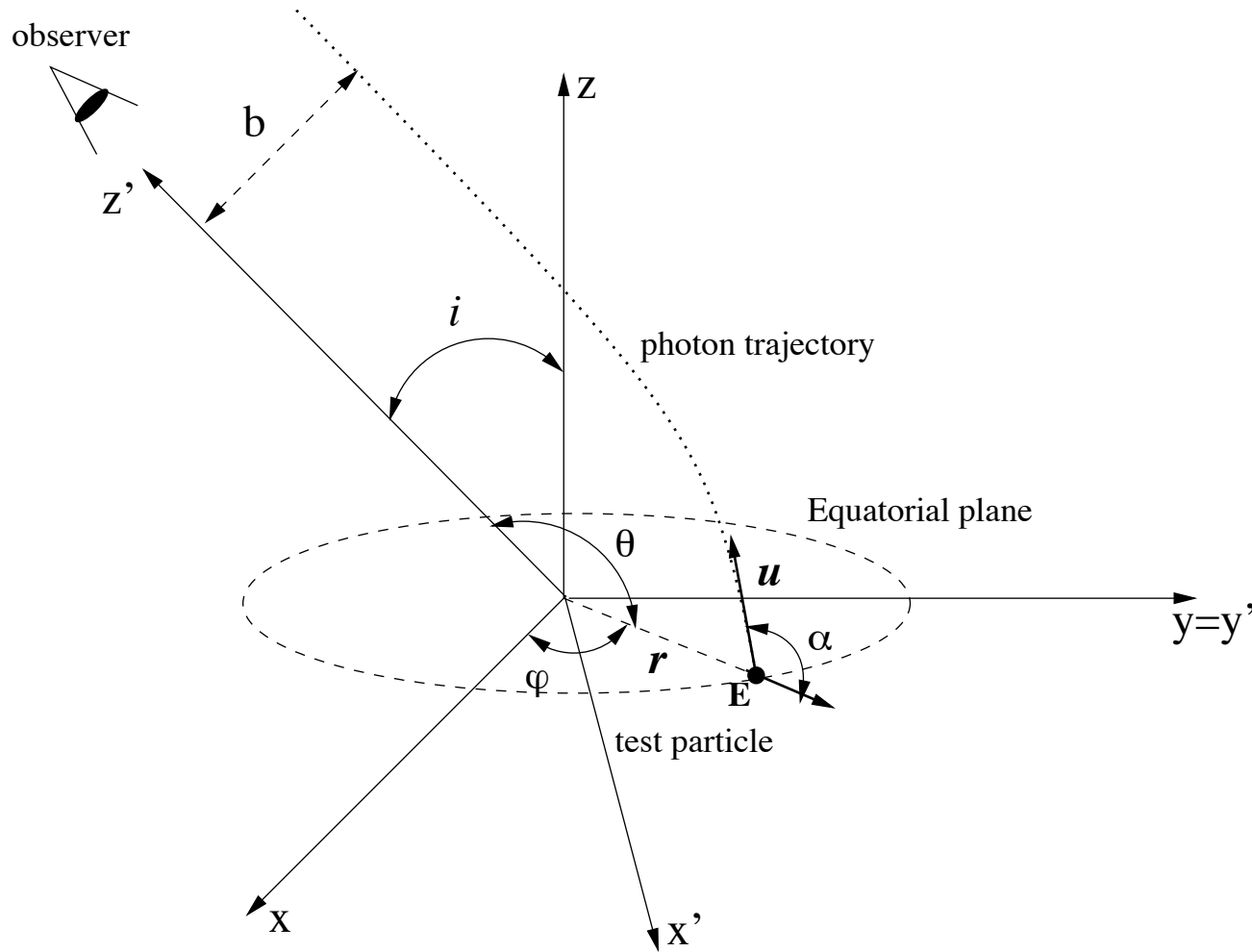
RELATIVISTIC POYNTING-ROBERTSON EFFECT

(1) presence of gravity

(2) presence of gravity
and radiation processes

Ray-tracing approach

We consider the case of *static and spherically symmetric geometries*



The formula of the impact parameter is

$$b = \frac{R \sin \alpha}{\sqrt{e^\nu(R)}}$$

The critical impact parameter b_c is

$$b_c = b(\alpha = \pi/2, R = r_{\text{ps}})$$

photonsphere

For $b > b_c$ photon reaches the observer's location otherwise not

SIMULATION PARAMETERS

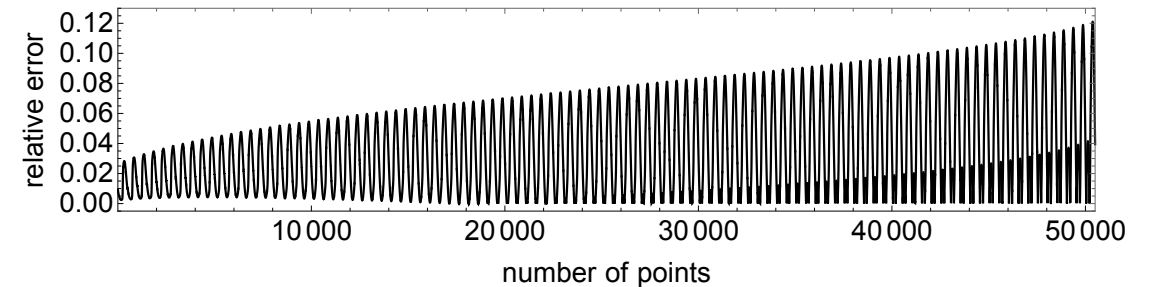
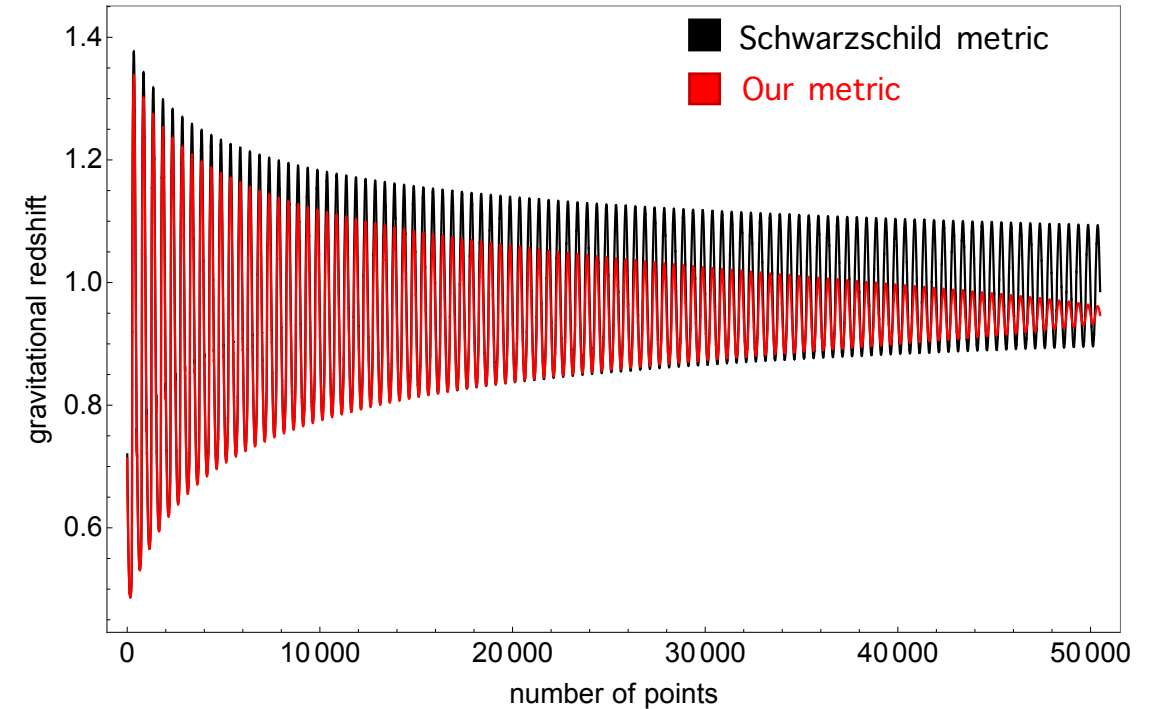
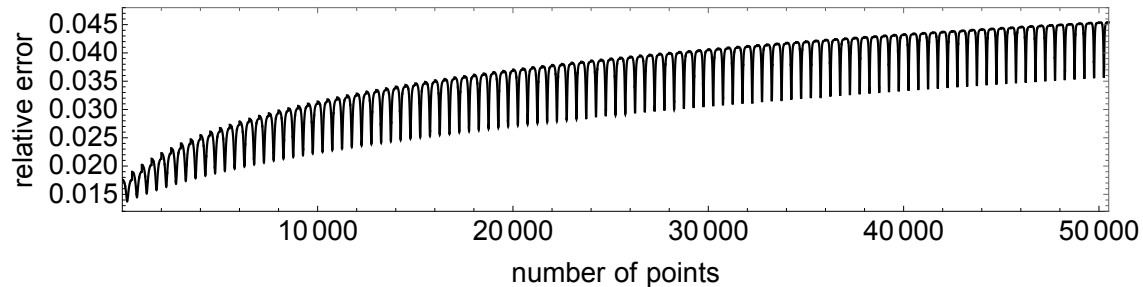
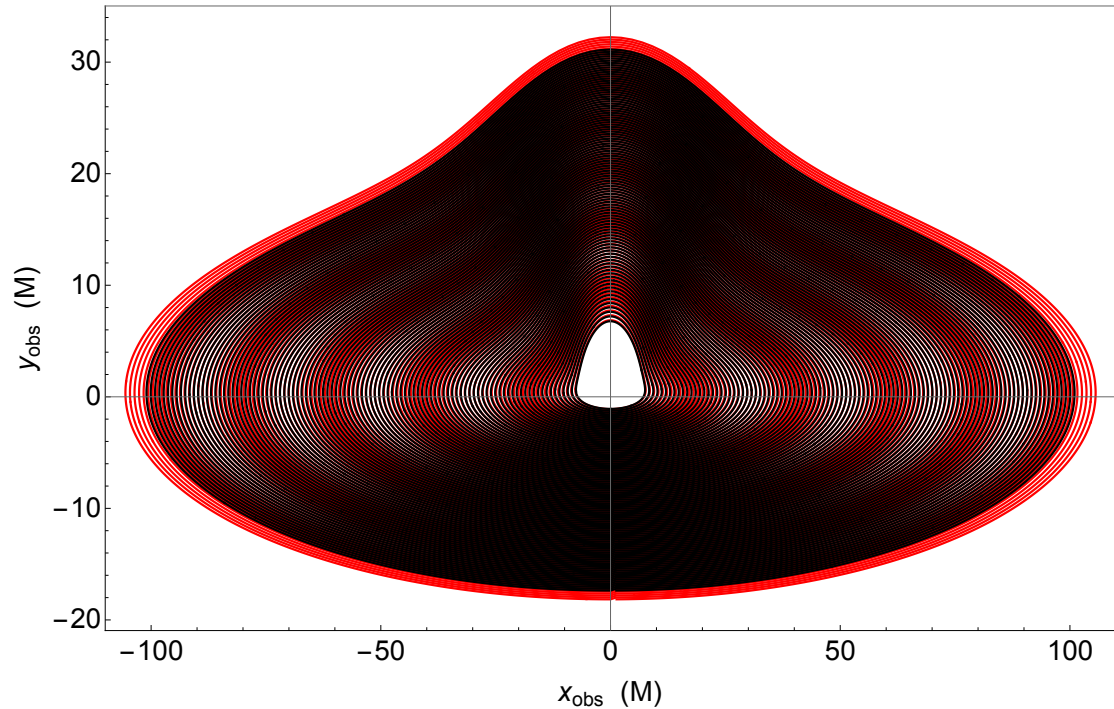
$i=80^\circ$

$r_{in}=6.23M$

$r_{out}=100M$

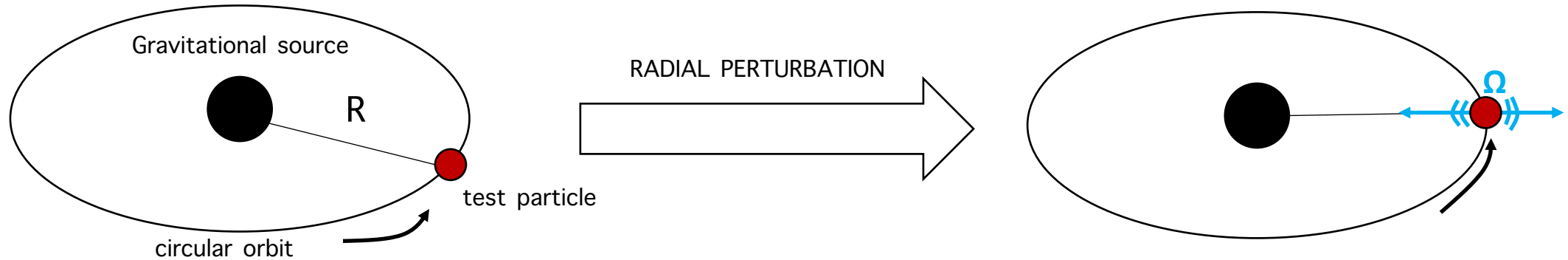
Image of a black hole

The matter in the accretion disk moves on Keplerian orbits.



Epicyclic frequencies

Let us consider the motion of a test particle influenced by an *isotropic radial force*



The epicyclic frequencies $\{\Omega_r, \Omega_\phi, \Omega_\theta\}$ have a strong dependence on the underlying *spacetime geometry* and are frequently observed in *X-ray binaries*. How to proceed for obtaining information on epicyclic frequencies?

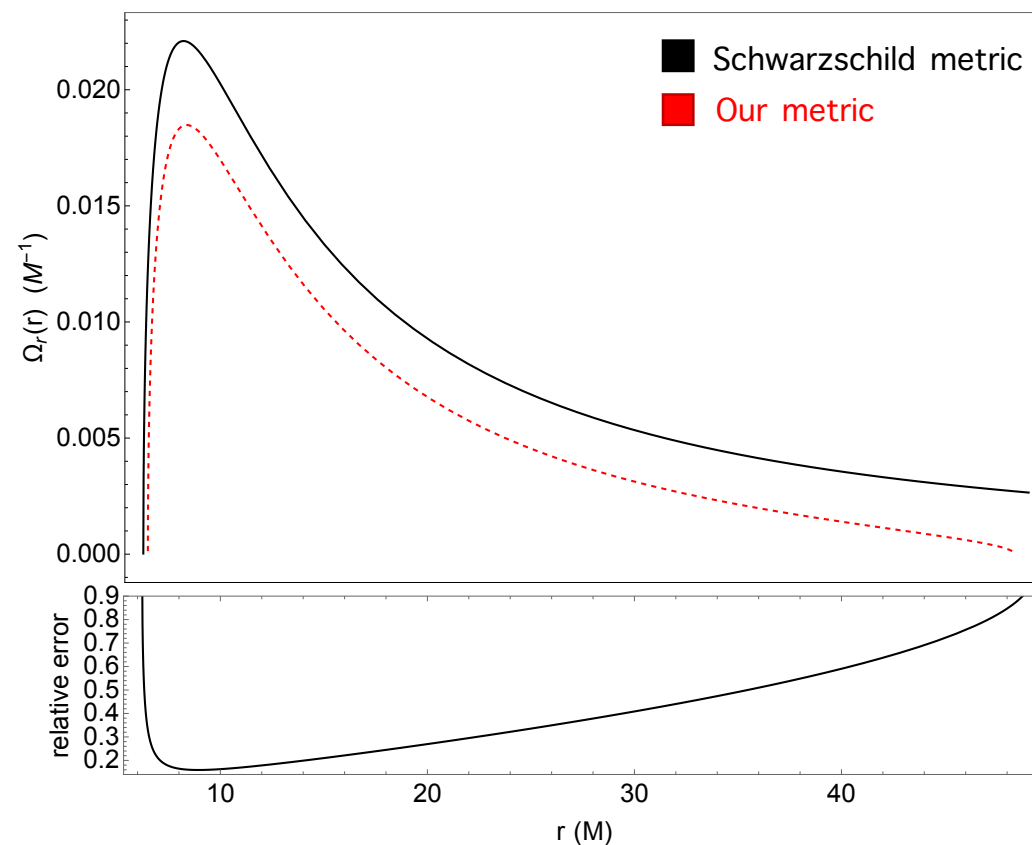
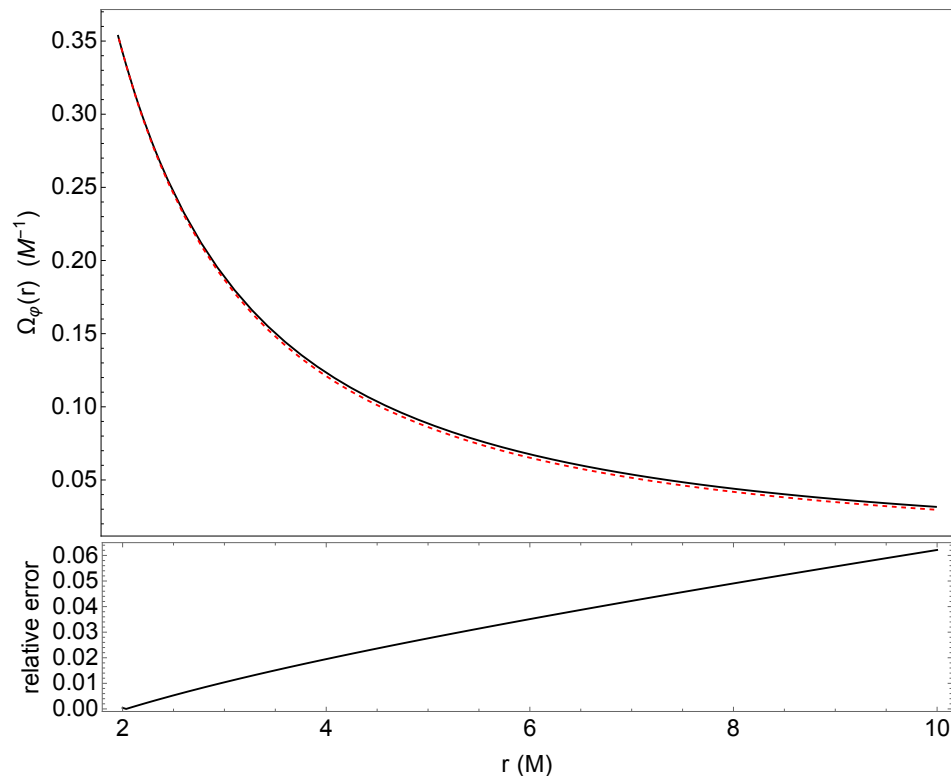


Epicyclic frequencies

The formula of the epicyclic frequencies are

$$\Omega_{\varphi} := \sqrt{\frac{-g'_{tt}(r)}{2r}} = \sqrt{\frac{e^{\nu(r)}\nu'(r)}{2r}},$$

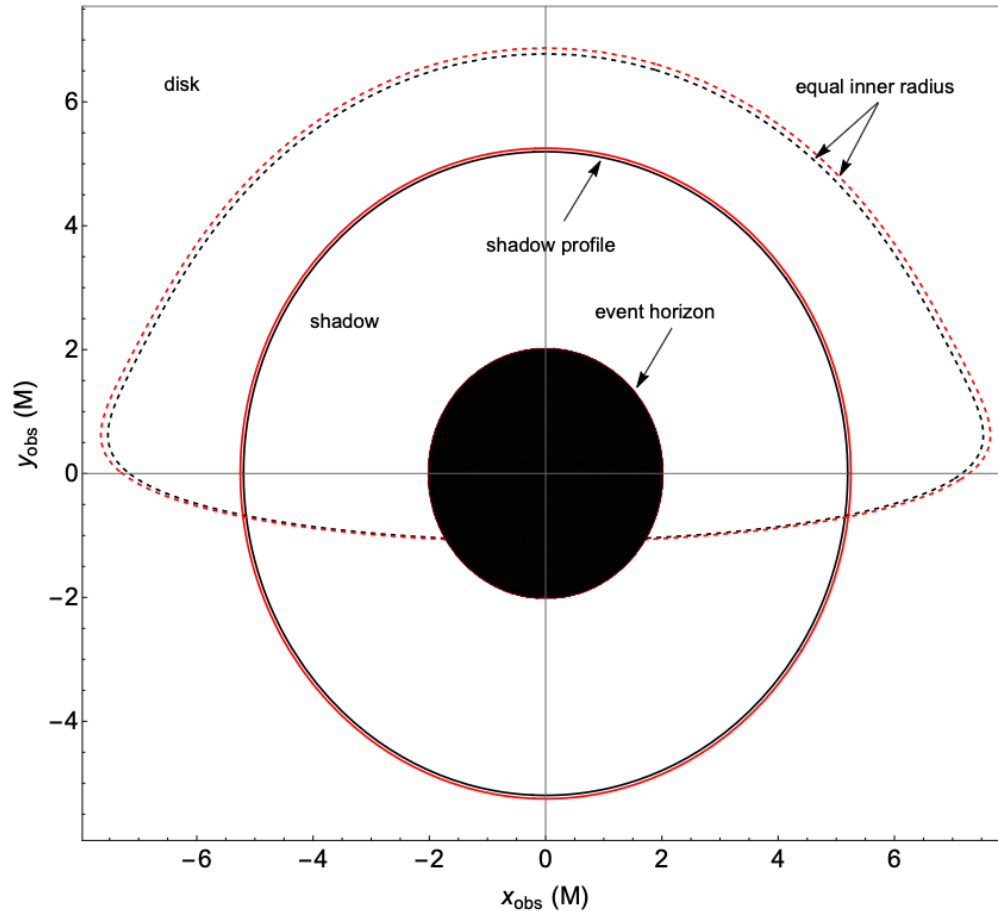
$$\Omega_r := \sqrt{\frac{g_{tt}^2(g^{tt})'' + 6\Omega_{\varphi}^2}{2g_{rr}}} = e^{\nu(r)} \sqrt{\frac{e^{-\nu(r)}[r\nu''(r) - r\nu'(r)^2 + 3\nu'(r)]}{2re^{\lambda(r)}}}.$$



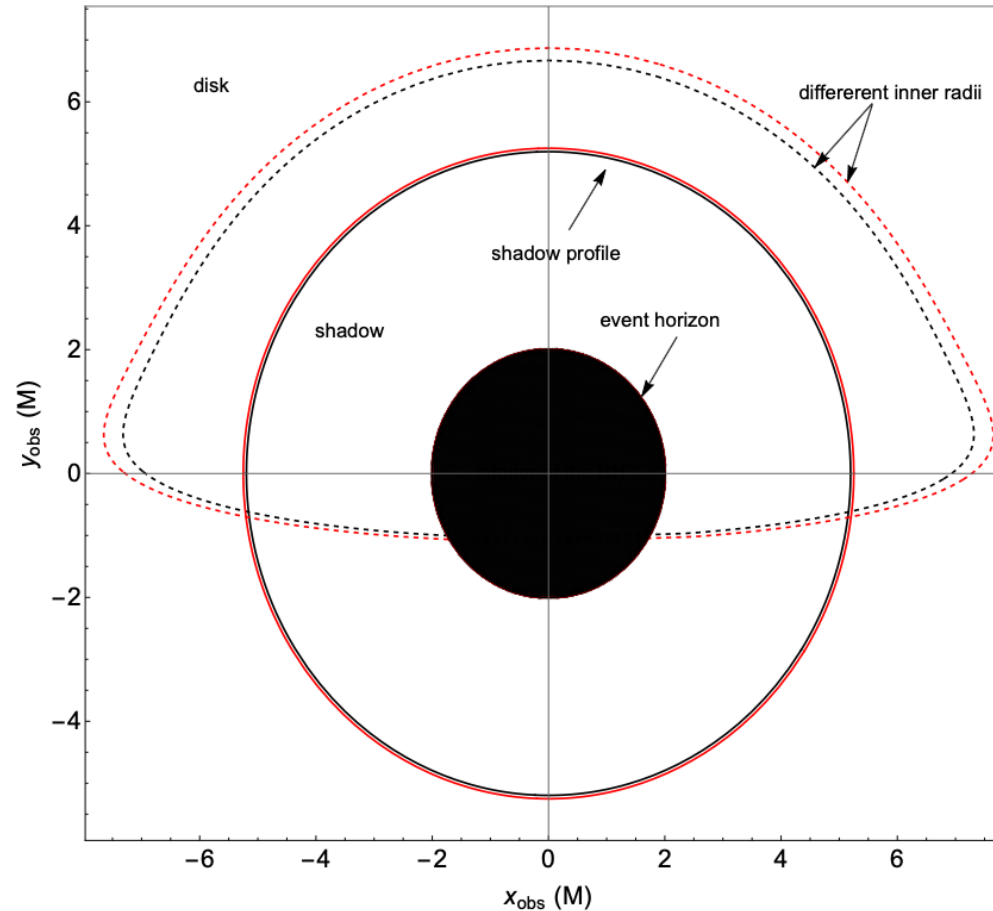
BH shadow profile

In spherically symmetric spacetimes the shadow profile reduces to a circle of radius b_c

SAME ISCO



DIFFERENT ISCO

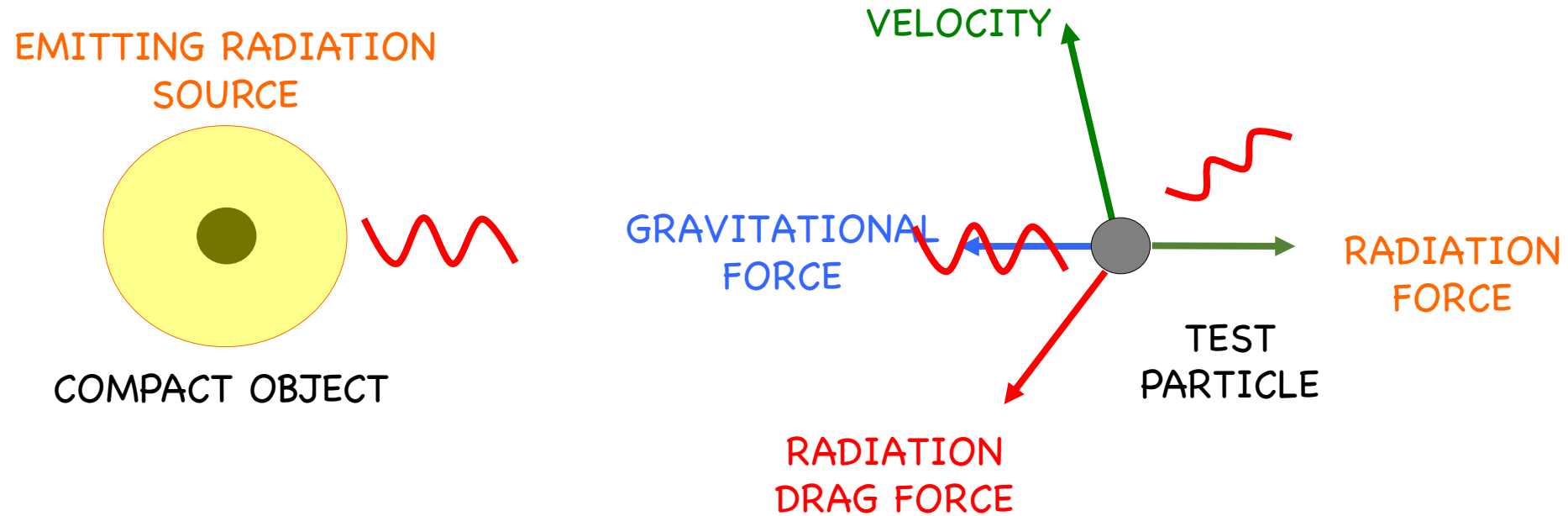


■ Schwarzschild metric
■ Our metric

$$b_c = 5.20M$$

$$b_c = 5.25M$$

General relativistic Poynting-Robertson (PR) effect



*action of the **electromagnetic radiation** on a moving body,
configuring as a **dissipative force***

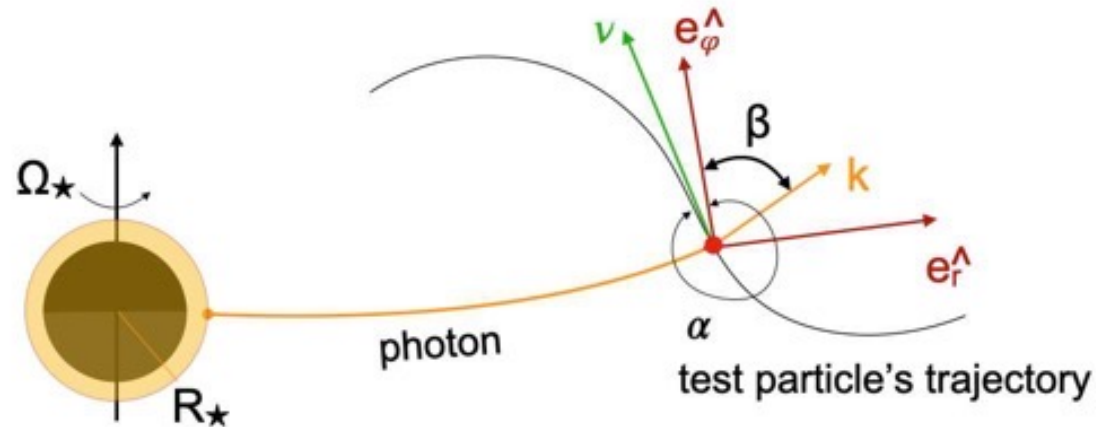
General relativistic Poynting-Robertson (PR) effect

The input parameters of this model in static and spherically symmetric metrics are

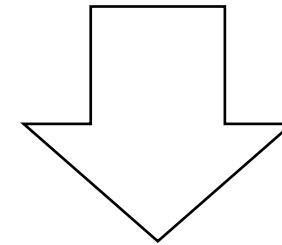
M
gravitational field

$A=(0.1, 0.4, 0.6)$ $R_{\star}=2.5M$ $\Omega_{\star}=(0, 0.09, 0.16) \text{ M}^{-1}$
 $A, b \longrightarrow R_{\star}, \Omega_{\star}$
radiation field

$r_{\text{ISCO}}\Omega_K(r_{\text{ISCO}})$
 $(\nu_0, \alpha_0, r_0, \varphi_0) = (\nu_K, 0, r_{\text{ISCO}}, 0)$
test particle's initial conditions



The PR effect has a *sensitive dependence from the initial conditions*

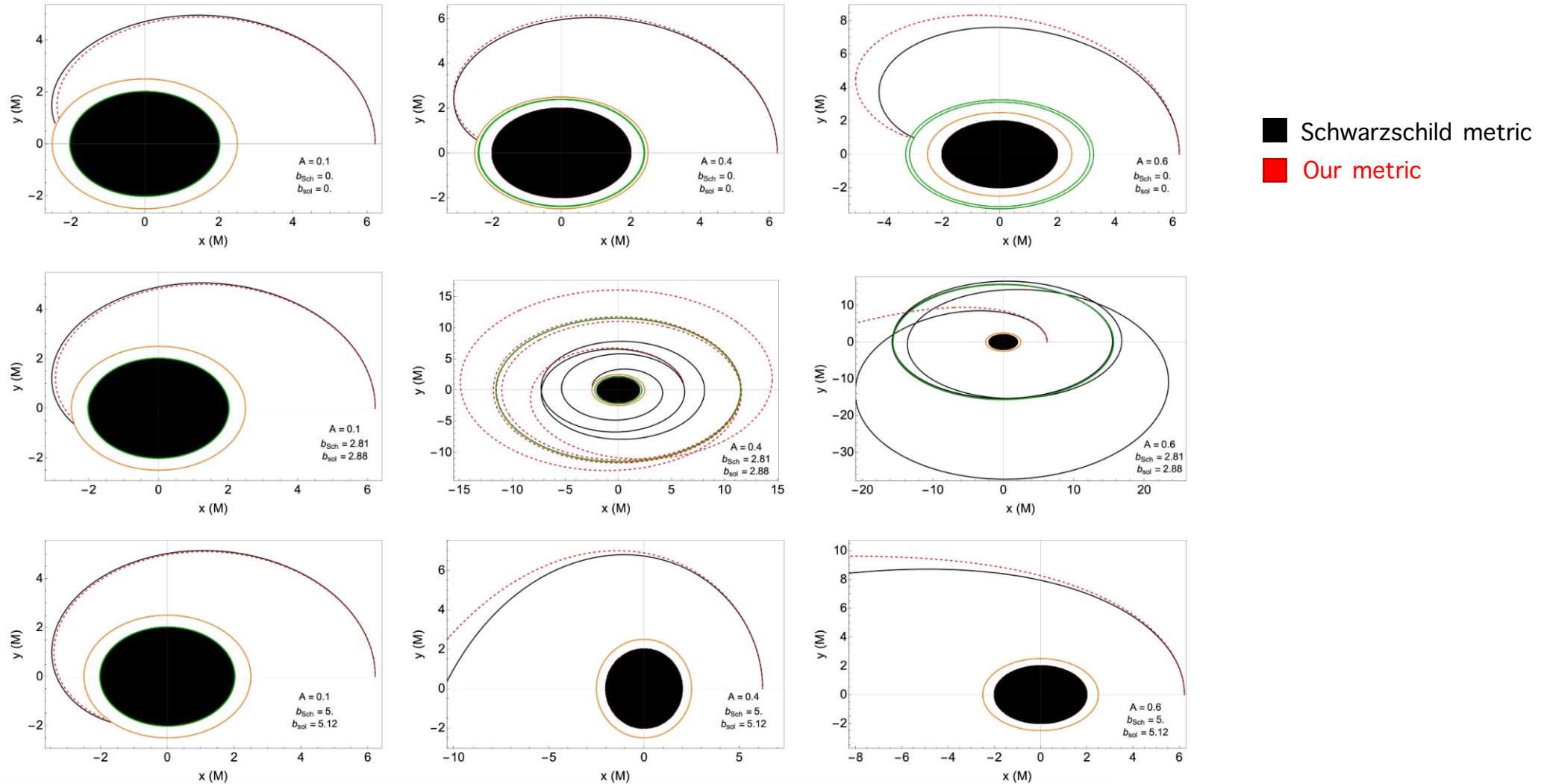


RELATIVISTIC POYNTING-ROBERTSON EFFECT

- (1) Different initial velocity
- (2) Different initial radius and velocity

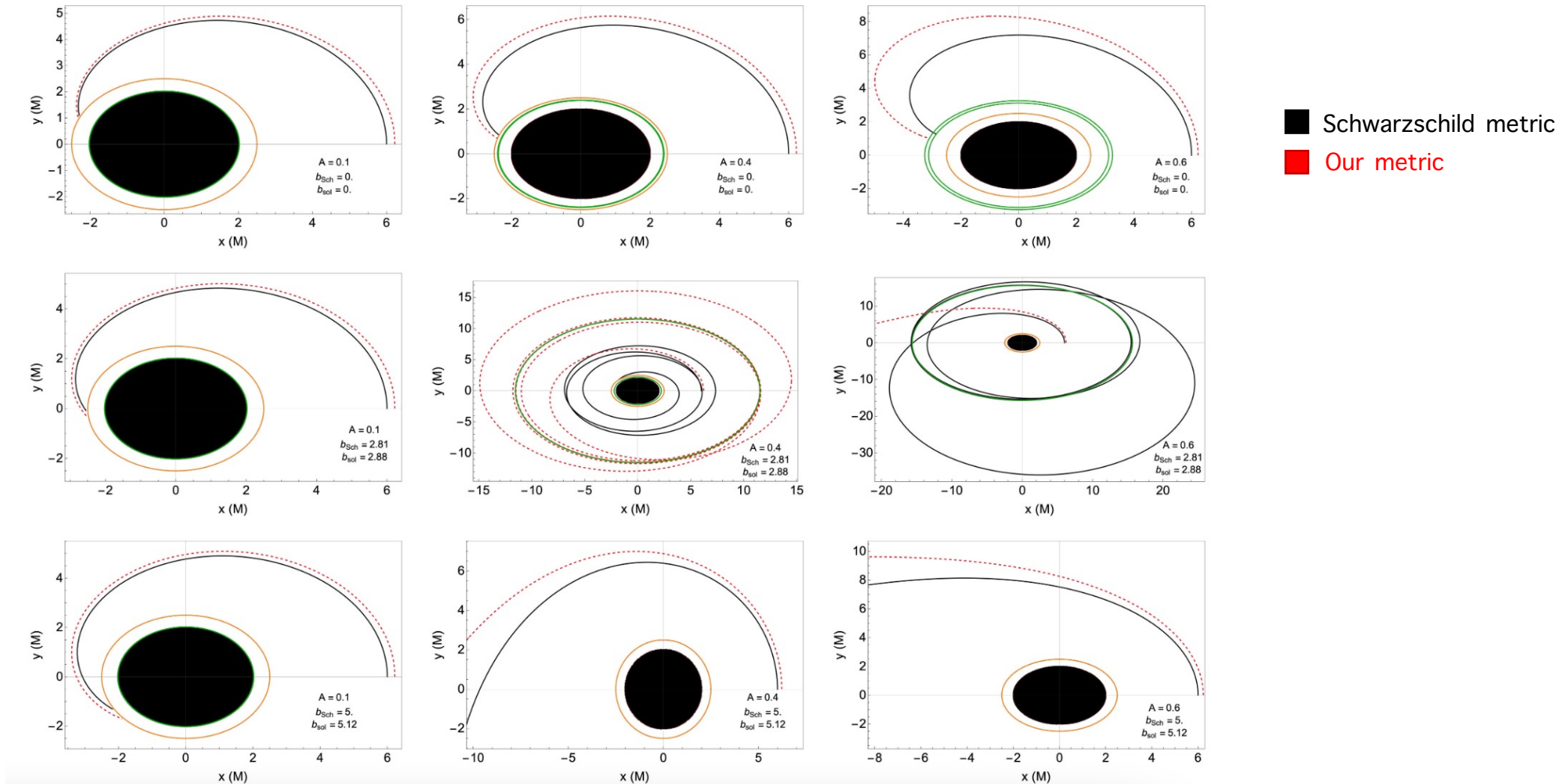
General relativistic Poynting-Robertson (PR) effect

(1) Different initial velocity



General relativistic Poynting-Robertson (PR) effect

(2) Different initial radius and velocity



Conclusions

ADAVANTAGES OF THE PROPOSED METHOD

Complementarity and full range gravitational regimes

Availability of data for epicyclic frequency

PR effect alone is a very powerful tool

LIMITS OF THE PROPOSED METHOD

Lack of sensitivity in EHT data

Having all data on the same source

FUTURE PERSPECTIVES

Include new astrophysical methods

Extension to stationary and axially symmetric metrics





THANK YOU
FOR YOUR
ATTENTION
