Modelling Magnetar Behaviour with 3D magnetothermal simulations

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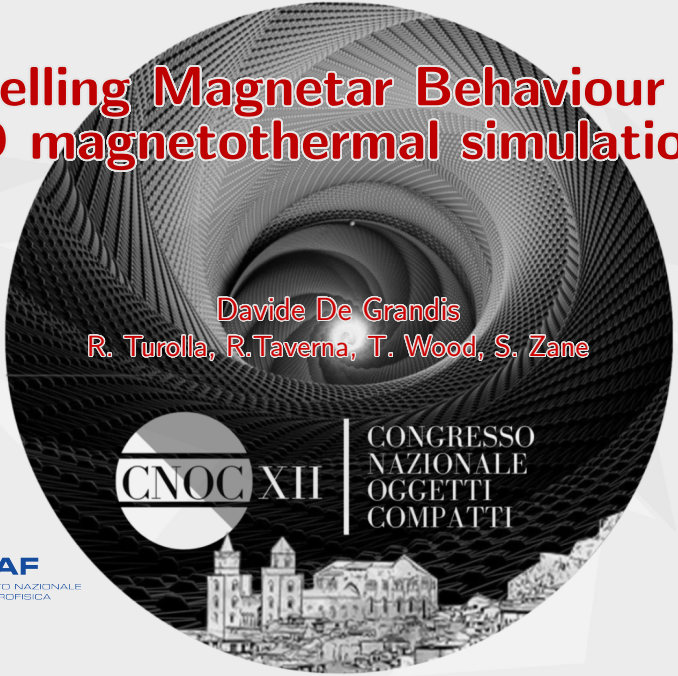

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Introduction

The observational properties of strongly magnetised NSs (magnetars) are dictated by their field:

- ☞ its topology determines heat conduction, shaping their thermal (X-ray) emission
- ☞ it powers violent, transient activity

Modelling the intertwined evolution of the $\boldsymbol{B}$-field and temperature is key in understanding magnetar behaviour

Different approaches have been developed in the last decades [Pons & Viganò 2019 for a review], but only recently *coupled magneto-thermal* and *3D* codes became available [De Grandis et al. 2020, Igoshev et al. 2020]

The Magneto-thermal Evolution equations

The solid crust is treated in the *eMHD regime*, in which only electrons can move. The governing equations are the *induction* and *heat balance* ones

conductivity current $\nabla \times \boldsymbol{B}/4\pi$ thermocouple

$$\frac{\partial \mathbf{B}}{\partial t} = -c\,\boldsymbol{\nabla} \times \Big[\underbrace{\sigma^{-1}\mathbf{J}}_{\text{Ohm term}} + \frac{1}{\mathrm{e}cn_e}\underbrace{\mathbf{J}\times\mathbf{B}}_{\text{Hall effect}} + \boldsymbol{G}\cdot\boldsymbol{\nabla}T \Big]$$

$$C_v\frac{\partial T}{\partial t} = -\boldsymbol{\nabla}\cdot\underbrace{\left(T\boldsymbol{G}\cdot\mathbf{J} - \boldsymbol{k}\cdot\boldsymbol{\nabla}T - \frac{\mu}{\mathrm{e}}\mathbf{J}\right)}_{\text{Heat flux}} + \underbrace{\mathbf{E}\cdot\mathbf{J}}_{\text{Joule heat}} - \underbrace{N_\nu}_{\text{neutrino loss}}$$

thermal conductivity

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In particular,the thermal conductivity is given by the Wiedemann-Franz for a *completely degenerate Fermi gas* of electrons

$$\boldsymbol{k} = \frac{\pi^2 k_{\mathrm{B}}^2 T}{3\mathrm{e}^2}\,\boldsymbol{\sigma} \quad \text{where} \quad (\boldsymbol{\sigma}^{-1})_{ij} = \sigma^{-1}\delta_{ij} + \frac{\epsilon_{ijk} B_k}{\mathrm{e}cn}$$

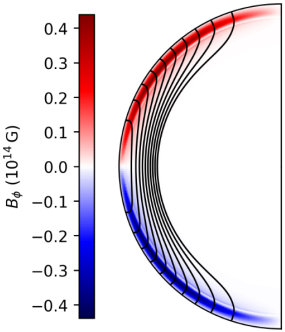
☞ heat flows preferentially along the field lines

Crustal Structure

We treat our equations in the *crust* only:

- ☞ we assume that Meissner effect completely expelled the magnetic field from the superconducting core
- ☞ the *inner crust* ($\rho \lesssim 10^{11}\,\mathrm{g\,cm^{-3}}$) controls the long term evolution
- ☞ the *outer crust* ($\rho \lesssim 10^{6}\,\mathrm{g\,cm^{-3}}$) has faster characteristic heat diffusion times $\Rightarrow$ controls short term phenomena
- ☞ the crust confinement of the field sets the BCs for the induction equation,

$$B_r(r_o) = 0 \;\&\; E_{\tan}(r_o) = 0; \qquad \nabla \times \boldsymbol{B}(r_\star) = 0$$



(crust width 4×)

From the Crust to the Surface: the outer BCs

The uppermost layers of a NS (*envelope*) are geometrically thin, but optically thick and host a huge temperature gradient. They are treated in plane-parallel approximation, and included in the simulation via the *Tsuruta law*

$$T_s(T_b, g, \boldsymbol{B}) = T_s^{(0)}(T_b, g)\, \mathcal{X}(T_b, \boldsymbol{B})$$

surface T

crustal T

gravity

transport through the envelope

field-aligned conduction

The relation is different for Fe-Ni or light-element (accreted) envelopes [see Beznogov et al. 2021]

Parody

These equations are solved by Parody, a Parallel FORTRAN code originally developed to study geomagnetism [Dormy 1997]

It is a so-called *pseudo-spectral* code, i.e.:

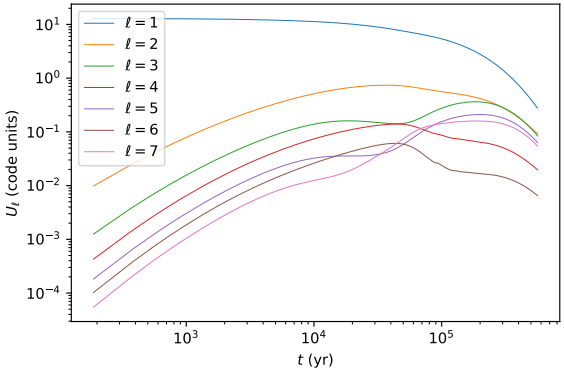
- ☞ uses a *finite difference* scheme for the radial part of the equation
- ☞ uses a *spectral* scheme for the angular part: all variables are expanded in spherical harmonics ⇒ angular differential operators algebrised

Time advance is treated partly with an implicit and partly with explicitly scheme, with Courant-condition preserving *adaptive time stepping*

The magnetic field in the Hall era

[see DDG et al. +20]

The Hall term transfer energies between poloidal and toroidal components and between different scales



An initially dipolar field cascades to a multipolar structure

Odd modes are favoured w.r.t. the nearby even ones

$\Downarrow$

The field tends to the *Hall attractor* [Gourgouliatos & Cumming 2014]

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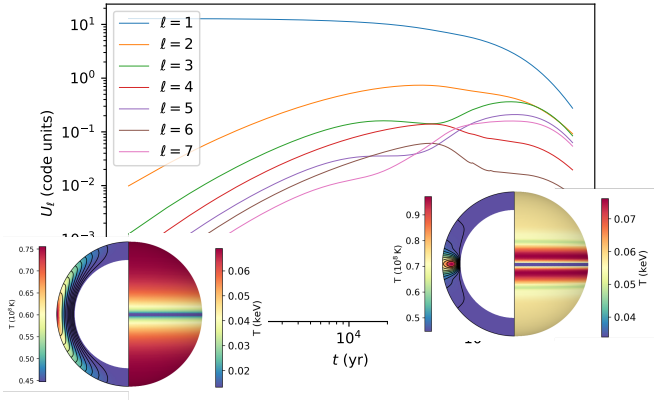

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⇓

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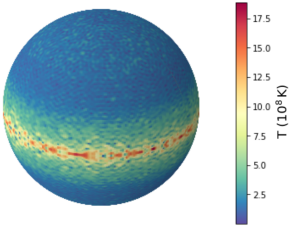
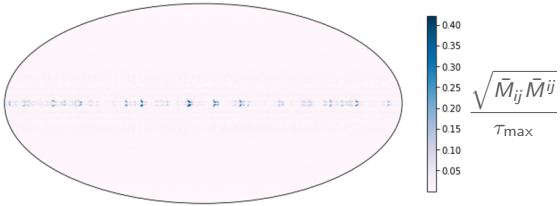
Magnetic Instabilities

[see DDG et al. +20]

Initially asymmetric fields can also create 3D structures via eMHD instabilities

Strong toroidal fields can trigger a *resistive tearing instability* [Wood et al. +14]





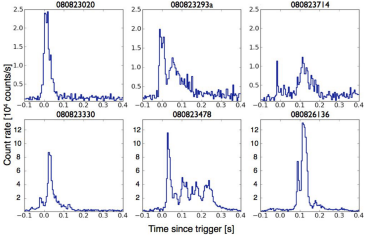
Small, hot structures with an enhanced magnetic stress are created

☞ The crust maximum yield [Chugunov & Horowitz +10] can be overcome, causing a failure

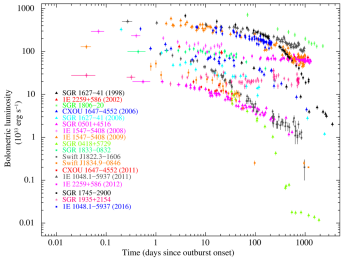
Magnetar activity

AXPs and SGRs are peculiar NSs characterised by violent X-ray activity

- ☞ *bursts*, lasting from $\approx 0.1\,\mathrm{s}$ to $\lesssim 100\,\mathrm{s}$ in the case of very energetic (but rare) giant flares, with a very abrupt rise and mostly thermal spectra;
- ☞ *outbursts* in which the X luminosity rises in $\approx$hours by a factor $100 - 1000\times$, to decay in $\approx 1\,\mathrm{y}$



short bursts from SGR 0501+4516 [from Huppenkothen et al., 2013]



a set of magnetar outbursts [Coti Zelati et al., 2018]

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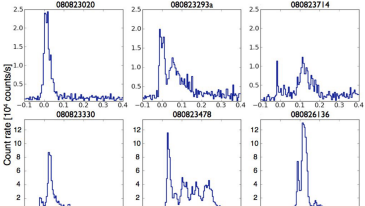
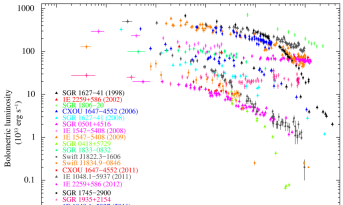




☞ They are believed to be powered by a *magnetar*, a NS powered by its (ultra-)strong magnetic field ($\approx 10^{14} - 10^{15}\,\mathrm{G}$)

Modelling Outbursts

Outburst spectra are typically described by two thermal components, with the hotter having a radius $\lesssim 1\,\mathrm{km}$

$\Downarrow$

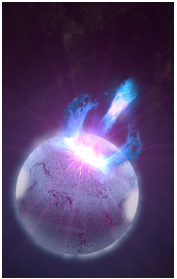
They are likely produced by a form of heat deposition in the crust

We model an outburst adding a heating term in a thin layer ($\lesssim 100\,\mathrm{m}$) in a patch in the outer crust ($\rho \approx 10^7\,\mathrm{g\,cm^{-3}}$)

$$C_V \frac{\partial T}{\partial t} = -\nabla \cdot \boldsymbol{Q} + \boldsymbol{E} \cdot \boldsymbol{J} - N_\nu(T) + s(r,\theta,\phi)\,\dot{H}'$$

for a small time Δt [cf. Pons & Rea 2012, Yakovlev et al. 2020]

The injection time was $\lesssim 1\,\mathrm{h}$, but the exact value is not important *as long as the total $H = \dot{H}\,\Delta t$ is the same*

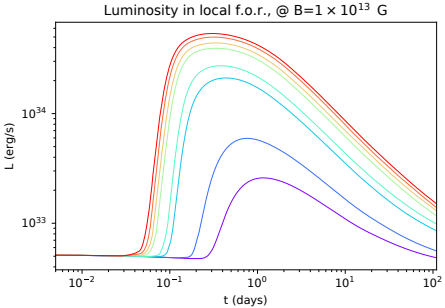


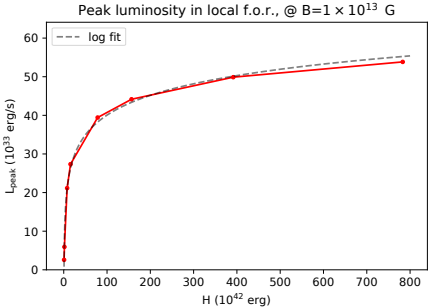
Neutrino Saturation

[see DDG et al. +22]

Luminosity sharply rises, then decreases over $\approx 100\,\mathrm{d}$

As the injected H increases, crustal neutrino emission is triggered and the luminosity saturates (as observed in 2D by [Pons & Rea 2012])





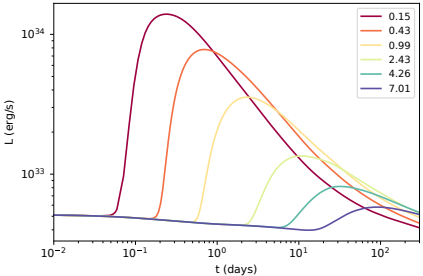
As the saturation L depends on the heated area, it's not a standard candle

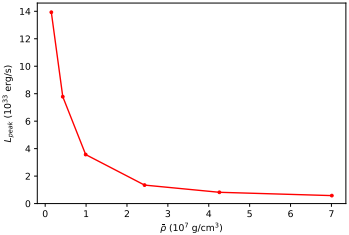
Dependence on the injection location

[see DDG et al. +22]

By injecting the same $H \simeq 3 \times 10^{40}$ erg deeper (i.e. at higher density), a smaller outburst magnitude is obtained

☞ higher heat capacity ☞ longer diffusion time for heat to reach the surface





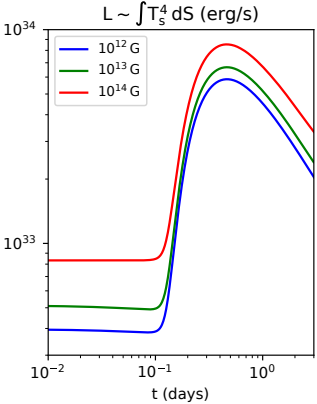
☞ Shallow heating is preferred (though parameter degeneracy is high!)

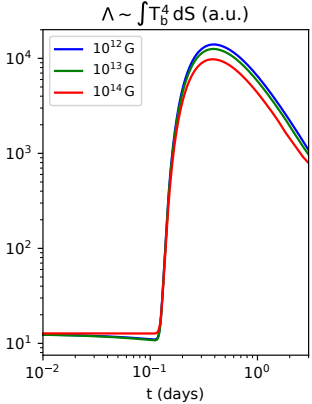
The role of the Magnetic Field I - Strength

[see DDG et al. +22]

Different strengths of the background dipolar field were considered

The field acts as an insulator, but it also affects the envelope heat conduction





Considering the same heat injection $H \simeq 3 \times 10^{40}$ erg in the same (polar) region, there is only a qualitative difference, while *the shape and timing is very similar*

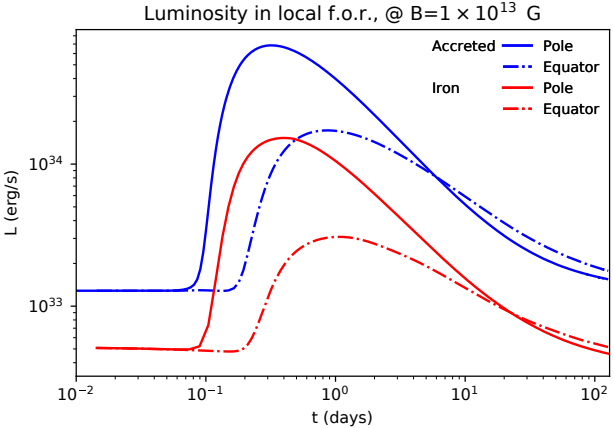
The role of the Magnetic Field I - Direction

[see DDG et al. +22]

The topology of the field in the heated region has a more direct impact

For the same $H \simeq 4 \times 10^{40}$ erg the outburst has a different profile for heating in different regions

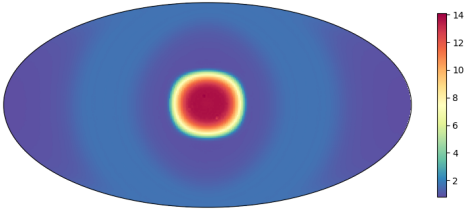
The more warped the field lines, the smaller and longer the burst



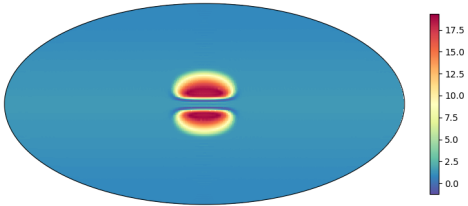
The role of the Magnetic Field II - Direction

[see DDG et al. +22]

The field geometry also affects the shape of the spot during the outburst rise: even if the injection has the same initial shape, heat transport follows field lines



(a) Polar injection (magnetic pole towards the observer, Fe envelope)



(b) Equatorial injection (magnetic pole upwards, Fe envelope)

Outbursts in a Multipolar Field

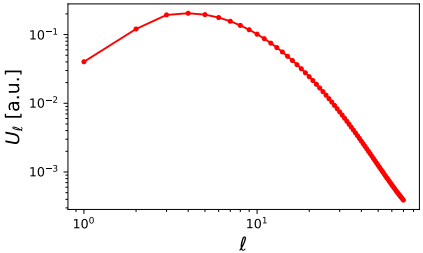
[see DDG et al. +22]

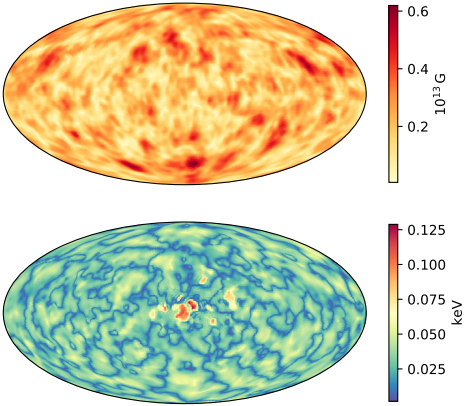
NS fields may go beyond the simple dipole

- ☞ "turbulent" field formation models
- ☞ observations of diverse thermal maps

Fields with a continuous spectrum produce a patchy thermal map [Igoshev et al. 2021]

Outburst spots get patchy, too, due to preferential routes for heat transport





Conclusions

- ☞ 3D, fully coupled magneto-thermal simulations are important to address the evolution of NSs, especially in the "Hall era"
- ☞ The study of NS thermal emission can give hints on the topology of their magnetic field via the thermal map structure
- ☞ Magnetar outbursts can be modelled as the result of a fast heat injection in the shallow layers (but it's hard to tell if it originates inside or above!)
- ☞ The magnetic topology affects directly the properties of the outbursts of transient magnetars

Thank you for your attention!